In[21]:=

```
 A = {{ax},{ay},{az}};
n = {{nx},{ny},{nz}};
X = {{x0},{y0},{z0}};
beta = {{u},{v}};
u0 = {{ux},{uy},{uz}};


t2 = Det[{
           Transpose[X ][[1]]
        ,  Transpose[n ][[1]]
        ,  Transpose[u0][[1]]  }];

v0 = Transpose [ {List[
            Coefficient[t2,x0]
        ,   Coefficient[t2,y0]
        ,   Coefficient[t2,z0]   ]} ];

replace_uz = Flatten[Solve[
           Flatten[Transpose[u0].n] == 0, uz]];

B = A + u u0 + v v0 /. replace_uz
```

Out[20]=

```
{{ax + u ux + (-(nz uy) + ny uz) v},
>    {ay + u uy + (nz ux - nx uz) v},
>    {az + u uz + (-(ny ux) + nx uy) v}} /. replace_uz
```

In[23]:=

```
 v0
```

Out[22]=

```
{{-(nz uy) + ny uz}, {nz ux - nx uz}, {-(ny ux) + nx uy}}
```

In[41]:=

```
replaceuz = (Solve[
           Flatten[Transpose[u0].n] == 0, uz] )
```

Out[40]=

$$\{\{uz \to -(\frac{nx\ ux + ny\ uy}{nz})\}\}$$

In[26]:=

```
%
```

Out[25]=

```
       nx ux + ny uy
{uz -> -(-------------)}
            nz
```

In[45]:=

```
B = Flatten[A + u u0 + v v0 /. replaceuz]
```

Out[44]=

```
                               ny (nx ux + ny uy)
{ax + u ux + (-(nz uy) - ------------------) v,
                                   nz
                               nx (nx ux + ny uy)
>    ay + u uy + (nz ux + ------------------) v,
                                   nz
        u (nx ux + ny uy)
>    az - ----------------- + (-(ny ux) + nx uy) v}
              nz
```

```
x = B[[1]]; y = B[[2]]; z = B[[3]];
xi = Simplify[- f/z x];  yi = Simplify[-f/z y];
```

In[52]:=

```
uvFromxy = Solve[ {xj == xi, yj == yi}, {u, v}]
```

Out[51]=

```
                      ax f nz + az nz xj
{{u -> -(-----------------------------) +
         f nz ux - nx ux xj - ny uy xj

                               2          2
>       ((-(f nx ny ux) - f ny  uy - f nz  uy - ny nz ux xj +
>            nx nz uy xj)
>          ((f nz ux - nx ux xj - ny uy xj)
>             (ay f nz + az nz yj) -
>            (ax f nz + az nz xj)
>             (f nz uy - nx ux yj - ny uy yj))) /
>        ((f nz ux - nx ux xj - ny uy xj)
                                    2          2
>          (-((-(f nx ny ux) - f ny  uy - f nz  uy -
>                 ny nz ux xj + nx nz uy xj)
>               (f nz uy - nx ux yj - ny uy yj)) +
>            (f nz ux - nx ux xj - ny uy xj)
                  2          2
>             (f nx  ux + f nz  ux + f nx ny uy -
>               ny nz ux yj + nx nz uy yj))),
>    v -> -(((f nz ux - nx ux xj - ny uy xj)
>            (ay f nz + az nz yj) -
>           (ax f nz + az nz xj)
>            (f nz uy - nx ux yj - ny uy yj)) /
                                   2          2
>         (-((-(f nx ny ux) - f ny  uy - f nz  uy -
>                ny nz ux xj + nx nz uy xj)
>              (f nz uy - nx ux yj - ny uy yj)) +
>           (f nz ux - nx ux xj - ny uy xj)
                 2          2
>            (f nx  ux + f nz  ux + f nx ny uy - ny nz ux yj +
>              nx nz uy yj)))}}
```

In[53]:=

```
Simplify[%]
```

Out[52]=

```
{{u -> (nz (ax f nx^2 ux + ay f nx ny ux + ax f nz^2 ux +
>          ax f nx ny uy + ay f ny^2 uy + ay f nz^2 uy +
>          az nx^2 ux xj + ay ny nz ux xj + az nz^2 ux xj +
>          az nx ny uy xj - ay nx nz uy xj + az nx ny ux yj -
>          ax ny nz ux yj + az ny^2 uy yj + ax nx nz uy yj +
>          az nz^2 uy yj)) /
>      ((nx^2 ux^2 + nz^2 ux^2 + 2 nx ny ux uy + ny^2 uy^2 +
>          nz^2 uy^2) (-(f nz) + nx xj + ny yj)),
>    v -> (nz (ay f nz ux - ax f nz uy - ay nx ux xj -
>          ay ny uy xj - az nz uy xj + ax nx ux yj +
>          az nz ux yj + ax ny uy yj)) /
>      ((nx^2 ux^2 + nz^2 ux^2 + 2 nx ny ux uy + ny^2 uy^2 +
>          nz^2 uy^2) (-(f nz) + nx xj + ny yj))}}
```

```
uv = %;
```

In[55]:=

```
ellipse = Simplify[
  (u^2/a^2 + v^2/b^2 == 1) /. uv
]
```

Out[54]=

```
{(nz^2 Power[ay f nz ux - ax f nz uy - ay nx ux xj -
>          ay ny uy xj - az nz uy xj + ax nx ux yj +
>          az nz ux yj + ax ny uy yj, 2]) /
>      (b^2 (nx^2 ux^2 + nz^2 ux^2 + 2 nx ny ux uy + ny^2 uy^2 +
>           nz^2 uy^2)^2 (-(f nz) + nx xj + ny yj)^2) +
>     (nz^2 Power[ax f nx^2 ux + ay f nx ny ux + ax f nz^2 ux +
>          ax f nx ny uy + ay f ny^2 uy + ay f nz^2 uy +
>          az nx^2 ux xj + ay ny nz ux xj + az nz^2 ux xj +
>          az nx ny uy xj - ay nx nz uy xj + az nx ny ux yj -
>          ax ny nz ux yj + az ny^2 uy yj + ax nx nz uy yj +
>          az nz^2 uy yj, 2]) /
>      (a^2 (nx^2 ux^2 + nz^2 ux^2 + 2 nx ny ux uy + ny^2 uy^2 +
>           nz^2 uy^2)^2 (-(f nz) + nx xj + ny yj)^2) == 1}
```